

Numerical errors in weight vector computation

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Weight vector computation problem

Minimize array output in all directions but that of the target direction \mathbf{d}

Minimization problem (1):



\mathbf{C} is signal covariance matrix

$$\text{Solution: } \mathbf{w} = \frac{\mathbf{C}^{-1}\mathbf{d}}{\mathbf{d}^H\mathbf{C}^{-1}\mathbf{d}}$$

C is approximated by

$$A = \sum S_k S_k^H = X^H X \quad \text{where } X = [S_1, S_2, \dots, S_K]$$

Minimization problem (2):



A is sample covariance matrix

$$\text{Solution: } w = \frac{A^{-1}d}{d^H A^{-1}d}$$

$A = X^H X$. Minimization problem (3):



constrained least squares

Algorithms for (2)

$A = X^H X > 0$, need to compute $A^{-1}d$

Normal Equations (NE)		Semi NE
<p>(I) Cholesky</p> <ol style="list-style-type: none"> 1) $A = U^H U$ 2) $U^H u = d$ 3) $U v = u$ 4) $w = v / (d^H v)$ 	<p>(II) GE</p> <ol style="list-style-type: none"> 1) $A = L^H U$ 2) $L^H u = d$ 3) $U v = u$ 4) $w = v / (d^H v)$ 	<p>(III) QR</p> <ol style="list-style-type: none"> 1) $X = QR$ 2) $R^H u = d$ 3) $R v = u$ 4) $w = v / (d^H v)$

Computed A may become indefinite
so use GE instead of Cholesky

Algorithms for (3) – Null Space Method

$$1) Q^H \begin{bmatrix} d^H \\ X \end{bmatrix} H = \begin{bmatrix} 0 & 0 \\ e_1^T & 0 \\ L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix}$$

Generalized
QL decomposition

$$2) L_{22}W_2 = -L_{21} \quad (L_{22} \text{ is better conditioned than } X)$$

$$3) w = H \begin{bmatrix} 1 & w_2^H \end{bmatrix}^H$$

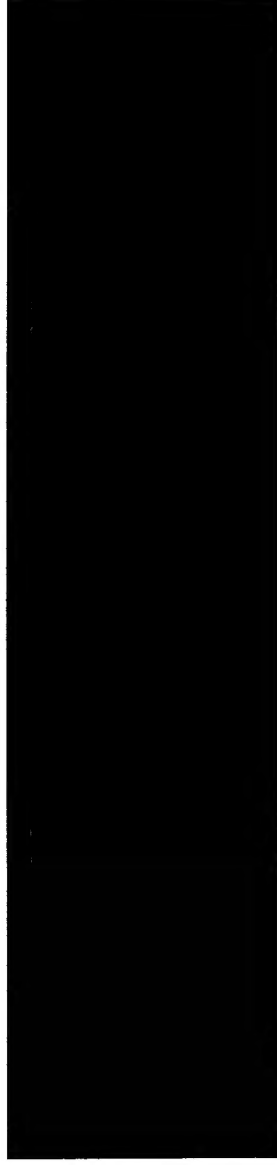
$$4) \|Xw\| = |L_{11}| \quad (L_{11} \text{ is the norm of the residual})$$

Sensitivity Analysis - NE

A replaced by $\hat{A} = A + \Delta A$, $\|\Delta A\| < \varepsilon \|A\|$

$$w = \frac{A^{-1}d}{d^H A^{-1}d} \quad \hat{w} = \frac{\hat{A}^{-1}d}{d^H \hat{A}^{-1}d}$$

Then



Sensitivity Analysis - SNE

X replaced by $\hat{X} = X + \Delta X$, $\|\Delta X\| < \varepsilon \|X\|$

$$w = \frac{(X^H X)^{-1} d}{d^H (X^H X)^{-1} d} \quad \hat{w} = \frac{(\hat{X}^H \hat{X})^{-1} d}{d^H (\hat{X}^H \hat{X})^{-1} d}$$

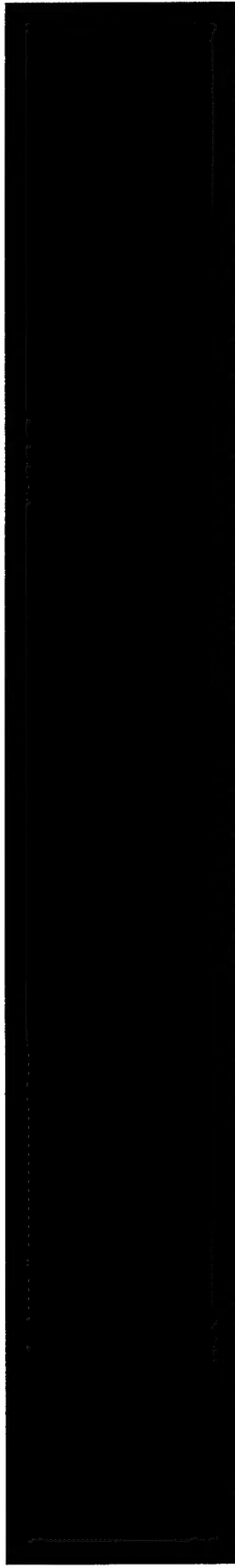


can be small

Sensitivity Analysis - NS

X replaced by $\hat{X} = X + \Delta X$, $\|\Delta X\| < \varepsilon \|X\|$

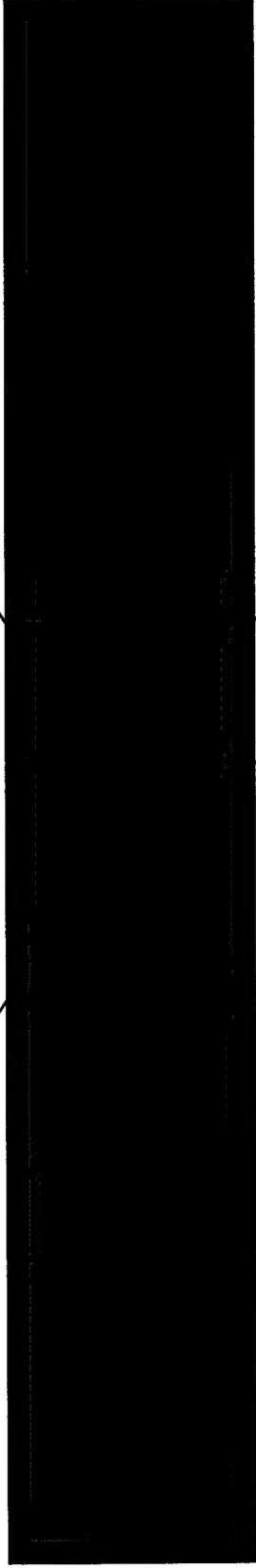
$$\min_{w^H d = 1} \|X w\|_2 \quad \min_{\hat{w}^H d = 1} \|\hat{X} \hat{w}\|_2$$



SNE&NS - Small residual case

$$X = W\Sigma V^H, \quad \Sigma = \text{diag}(\sigma_i), \quad V = [v_1, \dots, v_n]$$

If $d = v_n$ then $w = v_n$, $\|Xw\| = \sigma_n$ and



Conclusions

- SNE and NS are equally accurate
 - this is not the case for general LS problems
- If $\|Xw\| = \sigma_n$ then SNE and SN are more accurate than NE
 - in NE use GE instead of Cholesky
- If $\|Xw\| = \sigma_1$ then NE, SNE and SN are equal
 - NE is the least expensive
- cond number determined by submatrices of L
 - cond number can be small even if that of X is large